**Tutorial 9: Newton Polynomial Threshold Scheme**

a) Master key *K*=139

b) Threshold Scheme using Newton Polynomial mod a prime P = 257. The policy is *m* = 5 of *n* = 8 shadow keys. Given the master key K as *a*0. Set a polynomial of degree *m*−1 from coefficients

[*a*0, *a*1, *a*2, …, *am*−1] in Table 1 below.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *i* | 0 | 1 | 2 | 3 | 4 |
| *ai* | *K* | 19 | 23 | 29 | 43 |

Table 1: Secret coefficients of a master polynomial.

Generate *n*=8 shadow keys (*xi*. *yi*) for *i*= 0, 1, ..., *n*−1 from polynomial

A(*x*) = *a*0 + *a*1⋅*x* + *a*2⋅*x*2 +...+ *am*−1 ⋅ *xm*−1

From *x*0, *x*1, *x*2, ..., *xn*−1, compute *y*0, *y*1, *y*2, ..., *yn*−1 by *yi* =A(*xi*) mod P.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *i* | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| *xi* | 3 | 5 | 7 | 9 | 11 | 13 | 15 | 17 |
| *yi* | 43 | 212 | 224 | 121 | 9 | 58 | 245 | 97 |

Table 2: 8 Shadow keys are spread out.

c) Take the first 5 points of shadow key and build to a divided difference table.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *i* | *xi* | *yi* | *yi*′ | *yi*″ | *yi*‴ | *yi*(4) |
| 0 | 3 | *y*0 |  |  |  |  |
|  |  |  | *y*01 |  |  |  |
| 1 | 5 | *y*1 |  | *y*02 |  |  |
|  |  |  | *y*12 |  | *y*03 |  |
| 2 | 7 | *y*2 |  | *y*13 |  | *y*04 |
|  |  |  | *y*23 |  | *y*14 |  |
| 3 | 9 | *y*3 |  | *y*24 |  |  |
|  |  |  | *y*34 |  |  |  |
| 4 | 11 | *y*4 |  |  |  |  |

Table 3: Generate a friendly divided difference table.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *x*i | *y*i | |  | | --- | | nume | | deno |  | nume | deno |  | nume | deno |  | nume | deno |  |
| 3 | 43 |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  | -169 | 129 | 44 |  |  |  |  |  |  |  |  |  |
| 5 | 212 |  |  |  | -207 | 193 | 141 |  |  |  |  |  |  |
|  |  | -12 | 129 | 251 |  |  |  | 59 | 43 | 224 |  |  |  |
| 7 | 224 |  |  |  | 71 | 193 | 82 |  |  |  | 87 | 225 | 43 |
|  |  | 103 | 129 | 180 |  |  |  | 51 | 43 | 137 |  |  |  |
| 9 | 121 |  |  |  | 124 | 193 | 31 |  |  |  |  |  |  |
|  |  | 112 | 129 | 56 |  |  |  |  |  |  |  |  |  |
| 11 | 9 |  |  |  |  |  |  |  |  |  |  |  |  |

d) Generate a polynomial P*m*−1(*x*) via Newton interpolation.

P*m*−1(*x*) = *y*0

+ *y*01(*x*− *x*0)

+ *y*02(*x*− *x*0)(*x*− *x*1)

+ *y*03(*x*− *x*0)(*x*− *x*1)(*x*− *x*2)

+ *y*04(*x*− *x*0)(*x*− *x*1)(*x*− *x*2)(*x*− *x*3)

P4(*x*) = 43

+ 44 (*x*−3)

+ 141 (*x*−3)(*x*−5)

+ 224 (*x*−3)(*x*−5)(*x*−7)

+ 43 (*x*−3)(*x*−5)(*x*−7)(*x*−9)

e) Evaluate the polynomial P*m*−1(*x*) at *x* = 0 to regenerate the master key K.

P4(*x*) = 43

+ 44 (0−3)

+ 141 (0−3)(0−5)

+ 224 (0−3)(0−5)(0−7)

+ 43 (0−3)(0−5)(0−7)(0−9) = 123 (mod 257)

**Sample Solutions:**

A(*x*) = *a*0 + *a*1⋅*x* + *a*2⋅*x*2 +...+ *a*4 ⋅ *x*4

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| *x*0 | *x*1 | *x*2 | *x*3 | *x*4 | *x*5 | *x*6 | *x*7 |
| 4 | 173 | 185 | 82 | 227 | 19 | 206 | 58 |

Divided Difference Table

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *a*0 | *a*1 | *a*2 | *a*3 | *a*4 | *y*0 | *y*1 | *y*2 | *y*3 | *y*4 | *y*5 | *y*6 | *y*7 | *y*0 | *y*01 | *y*02 | *y*03 | *y*04 |
| 100 | 19 | 23 | 29 | 43 | 4 | 173 | 185 | 82 | 227 | 19 | 206 | 58 | 4 | 213 | 141 | 33 | 43 |
| 101 | 19 | 23 | 29 | 43 | 5 | 174 | 186 | 83 | 228 | 20 | 207 | 59 | 5 | 213 | 141 | 33 | 43 |
| 102 | 19 | 23 | 29 | 43 | 6 | 175 | 187 | 84 | 229 | 21 | 208 | 60 | 6 | 213 | 141 | 33 | 43 |
| 103 | 19 | 23 | 29 | 43 | 7 | 176 | 188 | 85 | 230 | 22 | 209 | 61 | 7 | 213 | 141 | 33 | 43 |
| 104 | 19 | 23 | 29 | 43 | 8 | 177 | 189 | 86 | 231 | 23 | 210 | 62 | 8 | 213 | 141 | 33 | 43 |
| 105 | 19 | 23 | 29 | 43 | 9 | 178 | 190 | 87 | 232 | 24 | 211 | 63 | 9 | 213 | 141 | 33 | 43 |
| 106 | 19 | 23 | 29 | 43 | 10 | 179 | 191 | 88 | 233 | 25 | 212 | 64 | 10 | 213 | 141 | 33 | 43 |
| 107 | 19 | 23 | 29 | 43 | 11 | 180 | 192 | 89 | 234 | 26 | 213 | 65 | 11 | 213 | 141 | 33 | 43 |
| 108 | 19 | 23 | 29 | 43 | 12 | 181 | 193 | 90 | 235 | 27 | 214 | 66 | 12 | 213 | 141 | 33 | 43 |
| 109 | 19 | 23 | 29 | 43 | 13 | 182 | 194 | 91 | 236 | 28 | 215 | 67 | 13 | 213 | 141 | 33 | 43 |
| 110 | 19 | 23 | 29 | 43 | 14 | 183 | 195 | 92 | 237 | 29 | 216 | 68 | 14 | 213 | 141 | 33 | 43 |
| 111 | 19 | 23 | 29 | 43 | 15 | 184 | 196 | 93 | 238 | 30 | 217 | 69 | 15 | 213 | 141 | 33 | 43 |
| 112 | 19 | 23 | 29 | 43 | 16 | 185 | 197 | 94 | 239 | 31 | 218 | 70 | 16 | 213 | 141 | 33 | 43 |
| 113 | 19 | 23 | 29 | 43 | 17 | 186 | 198 | 95 | 240 | 32 | 219 | 71 | 17 | 213 | 141 | 33 | 43 |
| 114 | 19 | 23 | 29 | 43 | 18 | 187 | 199 | 96 | 241 | 33 | 220 | 72 | 18 | 213 | 141 | 33 | 43 |
| 115 | 19 | 23 | 29 | 43 | 19 | 188 | 200 | 97 | 242 | 34 | 221 | 73 | 19 | 213 | 141 | 33 | 43 |
| 116 | 19 | 23 | 29 | 43 | 20 | 189 | 201 | 98 | 243 | 35 | 222 | 74 | 20 | 213 | 141 | 33 | 43 |
| 117 | 19 | 23 | 29 | 43 | 21 | 190 | 202 | 99 | 244 | 36 | 223 | 75 | 21 | 213 | 141 | 33 | 43 |
| 118 | 19 | 23 | 29 | 43 | 22 | 191 | 203 | 100 | 245 | 37 | 224 | 76 | 22 | 213 | 141 | 33 | 43 |
| 119 | 19 | 23 | 29 | 43 | 23 | 192 | 204 | 101 | 246 | 38 | 225 | 77 | 23 | 213 | 141 | 33 | 43 |
| 120 | 19 | 23 | 29 | 43 | 24 | 193 | 205 | 102 | 247 | 39 | 226 | 78 | 24 | 213 | 141 | 33 | 43 |
| 121 | 19 | 23 | 29 | 43 | 25 | 194 | 206 | 103 | 248 | 40 | 227 | 79 | 25 | 213 | 141 | 33 | 43 |
| 122 | 19 | 23 | 29 | 43 | 26 | 195 | 207 | 104 | 249 | 41 | 228 | 80 | 26 | 213 | 141 | 33 | 43 |
| 123 | 19 | 23 | 29 | 43 | 27 | 196 | 208 | 105 | 250 | 42 | 229 | 81 | 27 | 213 | 141 | 33 | 43 |
| 124 | 19 | 23 | 29 | 43 | 28 | 197 | 209 | 106 | 251 | 43 | 230 | 82 | 28 | 213 | 141 | 33 | 43 |
| 125 | 19 | 23 | 29 | 43 | 29 | 198 | 210 | 107 | 252 | 44 | 231 | 83 | 29 | 213 | 141 | 33 | 43 |
| 126 | 19 | 23 | 29 | 43 | 30 | 199 | 211 | 108 | 253 | 45 | 232 | 84 | 30 | 213 | 141 | 33 | 43 |
| 127 | 19 | 23 | 29 | 43 | 31 | 200 | 212 | 109 | 254 | 46 | 233 | 85 | 31 | 213 | 141 | 33 | 43 |
| 128 | 19 | 23 | 29 | 43 | 32 | 201 | 213 | 110 | 255 | 47 | 234 | 86 | 32 | 213 | 141 | 33 | 43 |
| 129 | 19 | 23 | 29 | 43 | 33 | 202 | 214 | 111 | 256 | 48 | 235 | 87 | 33 | 213 | 141 | 33 | 43 |
| 130 | 19 | 23 | 29 | 43 | 34 | 203 | 215 | 112 | 0 | 49 | 236 | 88 | 34 | 213 | 141 | 33 | 43 |
| 131 | 19 | 23 | 29 | 43 | 35 | 204 | 216 | 113 | 1 | 50 | 237 | 89 | 35 | 213 | 141 | 33 | 43 |
| 132 | 19 | 23 | 29 | 43 | 36 | 205 | 217 | 114 | 2 | 51 | 238 | 90 | 36 | 213 | 141 | 33 | 43 |
| 133 | 19 | 23 | 29 | 43 | 37 | 206 | 218 | 115 | 3 | 52 | 239 | 91 | 37 | 213 | 141 | 33 | 43 |
| 134 | 19 | 23 | 29 | 43 | 38 | 207 | 219 | 116 | 4 | 53 | 240 | 92 | 38 | 213 | 141 | 33 | 43 |
| 135 | 19 | 23 | 29 | 43 | 39 | 208 | 220 | 117 | 5 | 54 | 241 | 93 | 39 | 213 | 141 | 33 | 43 |
| 136 | 19 | 23 | 29 | 43 | 40 | 209 | 221 | 118 | 6 | 55 | 242 | 94 | 40 | 213 | 141 | 33 | 43 |
| 137 | 19 | 23 | 29 | 43 | 41 | 210 | 222 | 119 | 7 | 56 | 243 | 95 | 41 | 213 | 141 | 33 | 43 |
| 138 | 19 | 23 | 29 | 43 | 42 | 211 | 223 | 120 | 8 | 57 | 244 | 96 | 42 | 213 | 141 | 33 | 43 |
| 139 | 19 | 23 | 29 | 43 | 43 | 212 | 224 | 121 | 9 | 58 | 245 | 97 | 43 | 213 | 141 | 33 | 43 |
| 140 | 19 | 23 | 29 | 43 | 44 | 213 | 225 | 122 | 10 | 59 | 246 | 98 | 44 | 213 | 141 | 33 | 43 |
| 141 | 19 | 23 | 29 | 43 | 45 | 214 | 226 | 123 | 11 | 60 | 247 | 99 | 45 | 213 | 141 | 33 | 43 |
| 142 | 19 | 23 | 29 | 43 | 46 | 215 | 227 | 124 | 12 | 61 | 248 | 100 | 46 | 213 | 141 | 33 | 43 |
| 143 | 19 | 23 | 29 | 43 | 47 | 216 | 228 | 125 | 13 | 62 | 249 | 101 | 47 | 213 | 141 | 33 | 43 |
| 144 | 19 | 23 | 29 | 43 | 48 | 217 | 229 | 126 | 14 | 63 | 250 | 102 | 48 | 213 | 141 | 33 | 43 |
| 145 | 19 | 23 | 29 | 43 | 49 | 218 | 230 | 127 | 15 | 64 | 251 | 103 | 49 | 213 | 141 | 33 | 43 |
| 146 | 19 | 23 | 29 | 43 | 50 | 219 | 231 | 128 | 16 | 65 | 252 | 104 | 50 | 213 | 141 | 33 | 43 |
| 147 | 19 | 23 | 29 | 43 | 51 | 220 | 232 | 129 | 17 | 66 | 253 | 105 | 51 | 213 | 141 | 33 | 43 |
| 148 | 19 | 23 | 29 | 43 | 52 | 221 | 233 | 130 | 18 | 67 | 254 | 106 | 52 | 213 | 141 | 33 | 43 |
| 149 | 19 | 23 | 29 | 43 | 53 | 222 | 234 | 131 | 19 | 68 | 255 | 107 | 53 | 213 | 141 | 33 | 43 |
| 150 | 19 | 23 | 29 | 43 | 54 | 223 | 235 | 132 | 20 | 69 | 256 | 108 | 54 | 213 | 141 | 33 | 43 |
| 151 | 19 | 23 | 29 | 43 | 55 | 224 | 236 | 133 | 21 | 70 | 0 | 109 | 55 | 213 | 141 | 33 | 43 |
| 152 | 19 | 23 | 29 | 43 | 56 | 225 | 237 | 134 | 22 | 71 | 1 | 110 | 56 | 213 | 141 | 33 | 43 |
| 153 | 19 | 23 | 29 | 43 | 57 | 226 | 238 | 135 | 23 | 72 | 2 | 111 | 57 | 213 | 141 | 33 | 43 |
| 154 | 19 | 23 | 29 | 43 | 58 | 227 | 239 | 136 | 24 | 73 | 3 | 112 | 58 | 213 | 141 | 33 | 43 |
| 155 | 19 | 23 | 29 | 43 | 59 | 228 | 240 | 137 | 25 | 74 | 4 | 113 | 59 | 213 | 141 | 33 | 43 |
| 156 | 19 | 23 | 29 | 43 | 60 | 229 | 241 | 138 | 26 | 75 | 5 | 114 | 60 | 213 | 141 | 33 | 43 |
| 157 | 19 | 23 | 29 | 43 | 61 | 230 | 242 | 139 | 27 | 76 | 6 | 115 | 61 | 213 | 141 | 33 | 43 |
| 158 | 19 | 23 | 29 | 43 | 62 | 231 | 243 | 140 | 28 | 77 | 7 | 116 | 62 | 213 | 141 | 33 | 43 |
| 159 | 19 | 23 | 29 | 43 | 63 | 232 | 244 | 141 | 29 | 78 | 8 | 117 | 63 | 213 | 141 | 33 | 43 |
| 160 | 19 | 23 | 29 | 43 | 64 | 233 | 245 | 142 | 30 | 79 | 9 | 118 | 64 | 213 | 141 | 33 | 43 |
| 161 | 19 | 23 | 29 | 43 | 65 | 234 | 246 | 143 | 31 | 80 | 10 | 119 | 65 | 213 | 141 | 33 | 43 |
| 162 | 19 | 23 | 29 | 43 | 66 | 235 | 247 | 144 | 32 | 81 | 11 | 120 | 66 | 213 | 141 | 33 | 43 |
| 163 | 19 | 23 | 29 | 43 | 67 | 236 | 248 | 145 | 33 | 82 | 12 | 121 | 67 | 213 | 141 | 33 | 43 |
| 164 | 19 | 23 | 29 | 43 | 68 | 237 | 249 | 146 | 34 | 83 | 13 | 122 | 68 | 213 | 141 | 33 | 43 |
| 165 | 19 | 23 | 29 | 43 | 69 | 238 | 250 | 147 | 35 | 84 | 14 | 123 | 69 | 213 | 141 | 33 | 43 |
| 166 | 19 | 23 | 29 | 43 | 70 | 239 | 251 | 148 | 36 | 85 | 15 | 124 | 70 | 213 | 141 | 33 | 43 |
| 167 | 19 | 23 | 29 | 43 | 71 | 240 | 252 | 149 | 37 | 86 | 16 | 125 | 71 | 213 | 141 | 33 | 43 |
| 168 | 19 | 23 | 29 | 43 | 72 | 241 | 253 | 150 | 38 | 87 | 17 | 126 | 72 | 213 | 141 | 33 | 43 |
| 169 | 19 | 23 | 29 | 43 | 73 | 242 | 254 | 151 | 39 | 88 | 18 | 127 | 73 | 213 | 141 | 33 | 43 |
| 170 | 19 | 23 | 29 | 43 | 74 | 243 | 255 | 152 | 40 | 89 | 19 | 128 | 74 | 213 | 141 | 33 | 43 |
| 171 | 19 | 23 | 29 | 43 | 75 | 244 | 256 | 153 | 41 | 90 | 20 | 129 | 75 | 213 | 141 | 33 | 43 |
| 172 | 19 | 23 | 29 | 43 | 76 | 245 | 0 | 154 | 42 | 91 | 21 | 130 | 76 | 213 | 141 | 33 | 43 |
| 173 | 19 | 23 | 29 | 43 | 77 | 246 | 1 | 155 | 43 | 92 | 22 | 131 | 77 | 213 | 141 | 33 | 43 |
| 174 | 19 | 23 | 29 | 43 | 78 | 247 | 2 | 156 | 44 | 93 | 23 | 132 | 78 | 213 | 141 | 33 | 43 |
| 175 | 19 | 23 | 29 | 43 | 79 | 248 | 3 | 157 | 45 | 94 | 24 | 133 | 79 | 213 | 141 | 33 | 43 |
| 176 | 19 | 23 | 29 | 43 | 80 | 249 | 4 | 158 | 46 | 95 | 25 | 134 | 80 | 213 | 141 | 33 | 43 |
| 177 | 19 | 23 | 29 | 43 | 81 | 250 | 5 | 159 | 47 | 96 | 26 | 135 | 81 | 213 | 141 | 33 | 43 |
| 178 | 19 | 23 | 29 | 43 | 82 | 251 | 6 | 160 | 48 | 97 | 27 | 136 | 82 | 213 | 141 | 33 | 43 |
| 179 | 19 | 23 | 29 | 43 | 83 | 252 | 7 | 161 | 49 | 98 | 28 | 137 | 83 | 213 | 141 | 33 | 43 |
| 180 | 19 | 23 | 29 | 43 | 84 | 253 | 8 | 162 | 50 | 99 | 29 | 138 | 84 | 213 | 141 | 33 | 43 |
| 181 | 19 | 23 | 29 | 43 | 85 | 254 | 9 | 163 | 51 | 100 | 30 | 139 | 85 | 213 | 141 | 33 | 43 |
| 182 | 19 | 23 | 29 | 43 | 86 | 255 | 10 | 164 | 52 | 101 | 31 | 140 | 86 | 213 | 141 | 33 | 43 |
| 183 | 19 | 23 | 29 | 43 | 87 | 256 | 11 | 165 | 53 | 102 | 32 | 141 | 87 | 213 | 141 | 33 | 43 |
| 184 | 19 | 23 | 29 | 43 | 88 | 0 | 12 | 166 | 54 | 103 | 33 | 142 | 88 | 213 | 141 | 33 | 43 |
| 185 | 19 | 23 | 29 | 43 | 89 | 1 | 13 | 167 | 55 | 104 | 34 | 143 | 89 | 213 | 141 | 33 | 43 |
| 186 | 19 | 23 | 29 | 43 | 90 | 2 | 14 | 168 | 56 | 105 | 35 | 144 | 90 | 213 | 141 | 33 | 43 |
| 187 | 19 | 23 | 29 | 43 | 91 | 3 | 15 | 169 | 57 | 106 | 36 | 145 | 91 | 213 | 141 | 33 | 43 |
| 188 | 19 | 23 | 29 | 43 | 92 | 4 | 16 | 170 | 58 | 107 | 37 | 146 | 92 | 213 | 141 | 33 | 43 |
| 189 | 19 | 23 | 29 | 43 | 93 | 5 | 17 | 171 | 59 | 108 | 38 | 147 | 93 | 213 | 141 | 33 | 43 |
| 190 | 19 | 23 | 29 | 43 | 94 | 6 | 18 | 172 | 60 | 109 | 39 | 148 | 94 | 213 | 141 | 33 | 43 |
| 191 | 19 | 23 | 29 | 43 | 95 | 7 | 19 | 173 | 61 | 110 | 40 | 149 | 95 | 213 | 141 | 33 | 43 |
| 192 | 19 | 23 | 29 | 43 | 96 | 8 | 20 | 174 | 62 | 111 | 41 | 150 | 96 | 213 | 141 | 33 | 43 |
| 193 | 19 | 23 | 29 | 43 | 97 | 9 | 21 | 175 | 63 | 112 | 42 | 151 | 97 | 213 | 141 | 33 | 43 |
| 194 | 19 | 23 | 29 | 43 | 98 | 10 | 22 | 176 | 64 | 113 | 43 | 152 | 98 | 213 | 141 | 33 | 43 |
| 195 | 19 | 23 | 29 | 43 | 99 | 11 | 23 | 177 | 65 | 114 | 44 | 153 | 99 | 213 | 141 | 33 | 43 |
| 196 | 19 | 23 | 29 | 43 | 100 | 12 | 24 | 178 | 66 | 115 | 45 | 154 | 100 | 213 | 141 | 33 | 43 |
| 197 | 19 | 23 | 29 | 43 | 101 | 13 | 25 | 179 | 67 | 116 | 46 | 155 | 101 | 213 | 141 | 33 | 43 |
| 198 | 19 | 23 | 29 | 43 | 102 | 14 | 26 | 180 | 68 | 117 | 47 | 156 | 102 | 213 | 141 | 33 | 43 |
| 199 | 19 | 23 | 29 | 43 | 103 | 15 | 27 | 181 | 69 | 118 | 48 | 157 | 103 | 213 | 141 | 33 | 43 |
| 200 | 19 | 23 | 29 | 43 | 104 | 16 | 28 | 182 | 70 | 119 | 49 | 158 | 104 | 213 | 141 | 33 | 43 |